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# Clausius relation and Friedmann equation in FRW universe model

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ABSTRACT: It has been shown that Friedmann equation of FRW universe can be derived from the first law of thermodynamics in Einstein gravity, Gauss-Bonnet gravity, Lovelock gravity, scalar-tensor gravity and $f(R)$ gravity. Moreover, it was pointed out that the temperature of the apparent horizon can be obtained using the tunneling formalism for the corresponding observers defined by Kodama vector. In this article, we find that the energy flux through the apparent horizon can be determined by using the Kodama vector. This implies the fact that the Clausius relation and the first law of thermodynamics associated with the apparent horizon in FRW universe is relative to the Kodama observers. We illustrate the derivation of Friedmann equation, and also extend the study to the cases of Hořava-Lifshitz gravity and IR modified Hořava-Lifshitz gravity.

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## Contents

## 1. Introduction

Since the discovery of black hole thermodynamics[1, 2, 3] in the 1970s, it has been widely accepted that a deep relationship exists between gravity theories and thermodynamics. This relationship possibly offers a window to the nature of quantum gravity. In 1995, Jacobson[4] derived Einstein equations by demanding fundamental Clausius relations $\delta Q = T \, dS$ for all the local Rindler horizon through each spacetime point, with $\delta Q$ and $T$ interpreted as the energy flux and Unruh temperature seen by the accelerated observer. In the FRW model, it is shown that the Friedmann equation which describes the dynamics of the universe can be derived from the first law of thermodynamics associated with the apparent horizon, by assuming a temperature $T = \frac{1}{2\pi\tilde{r}_A}$ and the entropy $S = \frac{A}{4}$, where $\tilde{r}_A$ and $A$ are the radius and area of the apparent horizon[5]. The derivations are proven in Einstein gravity, Gauss-Bonnet gravity, Lovelock gravity. Later on, it is shown that the Friedmann equation can also be regarded as the first law of thermodynamics in scalar-tensor gravity and f(R) gravity, by properly defining the energy density $\tilde{\rho}$ and pressure $\tilde{p}$[6]. It is further pointed out in [7] that the temperature $T = \frac{1}{2\pi\tilde{r}_A}$ associated with the apparent horizon can be obtained using the tunneling formalism. Thus an observer inside the apparent horizon will see a thermal spectrum when particles tunneling from outside the apparent horizon to inside the apparent horizon. These tunneling particles are detected by a Kodama observer, i.e., the energy of the tunneling particle is defined by using of the Kodama vector[8]. In this paper we propose a more direct way to calculate the energy flux $\delta Q$ across the apparent horizon, using the Kodama vector. This derivation has significant physical meanings. Combined with the derivation of temperature in [7] , it is strongly suggested that the first law of thermodynamics associated with the apparent horizon in the FRW model is relative to the "Kodama observer". This result enriches the discussion of thermodynamics in FRW universe. We also suggest to calculate the energy flux across the apparent horizon in this direct method. Then, using the temperature $T = \frac{1}{2\pi\tilde{r}_A}$ corresponding with the Kodama

vector and assuming the entropy to be a quarter of apparent horizon area, the Clausius relation $\delta Q = T\,dS$ implies the Friedmann equation of the FRW universe model.

Discussions on thermodynamics of the FRW universe model in various gravities theories and brane-world scenarios also refer to [9, 10, 11, 12, 13, 14]. A modified Friedmann equation can be derived from a quantum corrected area-entropy relation[15]. Recently, a new theory of gravity at a Lifshitz point was proposed by Hořava[16]. It may be regarded as a UV complete candidate for general relativity. The Hořava-Lifshitz theory has been intensively investigated[17, 18, 19, 20], and its application to cosmology has been studied[21, 22, 23]. Spherical symmetrical black hole solutions and cosmological solutions have been found[24, 25, 26, 27]. Since the generic IR vacuum of this theory is anti-de Sitter, in order to get a Minkowski vacuum in the IR, one should add a term $\mu^4 R$ into the action and take the $\Lambda_W \to 0$ limit. Black hole and cosmological solutions of this so called "IR modified Hořava-Lifshitz gravity" have also been studied[28, 29, 30, 31, 32, 33]. It is shown in a recent paper[34] that at the black hole horizon in Hořava-Lifshitz gravity, gravitational field equation can be casted into the form of the first law of thermodynamics. It is quite necessary to extend the study of relations between Friedmann equation and thermodynamics of FRW model to the cases of Hořava-Lifshitz gravity. In this article, we study the cosmological solutions of Hořava-Lifshitz gravity and IR modified Hořava-Lifshitz gravity, and show that Friedmann equation of FRW universe model in the Hořava theory can also be written in a form of first law of thermodynamics.

This paper is organized as follows. In Section 2 we review the derivation of Friedmann equation from the first law of thermodynamics in the context of Einstein gravity. We propose the direct method to calculate the energy flux across the apparent horizon, and discuss its physical meanings. In Section 3 we show that Friedmann equation of the FRW cosmological solutions of Hořava and IR modified Hořava gravity can also be written into the form of first law of apparent horizon thermodynamics. Section 4 is for conclusions and discussions.

## 2. First law of thermodynamics and Friedmann equation

In this section, we briefly review the first law of thermodynamics of the FRW metric, and give our calculation of the energy flux across the apparent horizon using the Kodama vector. The homogeneous and isotropic universe model is described by the $(n+1)$ dimensional Friedmann-Lemaitre-Robertson-Walker metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_{n-1}^2\right),\tag{2.1}$$

where $d\Omega_{n-1}^2$ denotes the line element of an (n-1)-dimensional unit sphere and the spatial curvature constant $k = +1, 0$ and $-1$ corresponding to a closed, flat and open universe, respectively. Defining $\tilde{r} = a(t)\,r$, the metric (2.1) can be rewritten as

$$ds^2 = h_{ab}dx^a dx^b + \tilde{r}^2 d\Omega_{n-1}^2,\tag{2.2}$$

where $x^0 = t$, $x^1 = r$, $h_{ab} = diag(-1, a^2/(1-kr^2))$. The dynamical apparent horizon $\mathcal{H}$ is a marginally trapped surface with vanishing expansion, determined by $h^{ab}\partial_a\tilde{r}\partial_b\tilde{r} = 0$,

which gives

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}\,, \tag{2.3}$$

where $H \equiv \dot{a}/a$ denotes the Hubble parameter. Also notice that the identity $\sqrt{1 - kr^2}\Big|_{\tilde{r}=\tilde{r}_A} = H\tilde{r}_A$ is satisfied.

Suppose that the energy-momentum tensor of the matter in the universe has the form of a perfect fluid $T_{ab} = (\rho + p)U_a U_b + pg_{ab}$, where $U^a$ denotes the four-velocity of the fluid, and $\rho$, $p$ are the energy density and pressure, respectively. In the FRW metric, the components of $T_{ab}$ are $T_{00} = \rho$, $T_{ij} = pg_{ij}$. Conservation law $\nabla_a T^{ab} = 0$ implies

$$\dot{\rho} + nH\left(\rho + p\right) = 0\,, \tag{2.4}$$

and the 00 component of the Einstein equation is the standard Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{16\pi G}{n\left(n - 1\right)}\rho\,, \tag{2.5}$$

which describes the dynamical evolution of the universe model.

It was proven in [5] that the Friedmann equation (2.5) can be derived from the first law of thermodynamics. They assumed that the apparent horizon has an associated entropy and temperature,

$$S = \frac{A}{4G}\,, \quad T = \frac{1}{2\pi\tilde{r}_A}\,, \tag{2.6}$$

where $A = n\Omega_n \tilde{r}_A^{n-1}$ is the area of the apparent horizon. According to [35, 36, 5], the first law of thermodynamics is rewritten as

$$d\mathcal{M} = A\Psi + WdV \tag{2.7}$$

where $V = \Omega_n \tilde{r}^n$ is the volume surrounded by the apparent horizon, and

$$\mathcal{M} = \frac{n(n-1)\Omega_n}{16\pi G}\tilde{r}^{n-2}(1 - h^{ab}\partial_a\tilde{r}\partial_b\tilde{r}) \tag{2.8}$$

is the Misner-Sharp energy[37], which can be identified as the total energy inside the apparent horizon. Following the discussion in[5], the work density

$$W = -\frac{1}{2}T^{ab}h_{ab}\,,$$

is regarded as the work done by a change of the apparent horizon. The energy-supply term

$$\Psi_a = T_a^{\ b}\partial_b\tilde{r} + W\partial_a\tilde{r}$$

determines the total energy flow $\delta Q = A\Psi$ through the apparent horizon.

The first law of thermodynamics in FRW model has been extensively discussed, however, a direct calculation of the energy flux $\delta Q$ from the observer viewpoint is still lacking. Here we shall propose a simpler way of calculating the energy flux $\delta Q$ by using the Kodama vector. It was shown in [7] that there is indeed a Hawking radiation with such a

temperature (2.6) of the apparent horizon by using the tunneling approach. It is the first time that the temperature of apparent horizon in FRW model can be illustrated by another independent method. In their proof, the energy of the particles tunneling through the apparent horizon is defined using the Kodama vector[8]. In other words, the temperature $T$ is "detected" by a Kodama observer. This result strongly suggests that the energy flux should be "detected" by the Kodama observer. The Kodama vector corresponding to metric (2.2) is defined as

$$K^a = -\epsilon^{ab}\nabla_b \tilde{r} = -\sqrt{1-kr^2}\left[-\left(\frac{\partial}{\partial t}\right)^a + Hr\left(\frac{\partial}{\partial r}\right)^a\right],$$ (2.9)

where $\epsilon_{ab} = a(t)/\sqrt{1-kr^2}(dt)_a \wedge (dr)_b$ [38]. The Kodama vector is very similar to the Killing vector $(\partial/\partial t)^a$ in the de Sitter space. In the stationary black hole spacetime, the timelike Killing vector can be used to define a conserved mass (energy). Since there is no timelike Killing vector in the dynamical black hole and FRW spacetime, the Kodama vector generates a preferred flow of time and is a dynamic analogue of a stationary Killing vector[39]. By the Kodama vector, a conserved quantity, Misner-Sharp energy[37], can be defined for the FRW spacetime[40]. The use of Kodama vector field as a preferred time evolution vector field in spherically symmetric dynamical systems also rises simplifications[41].

Now let's calculate the energy flux through the apparent horizon. We assume that all the energy flow across the apparent horizon is described by the perfect fluid $T_{ab}$. Since the temperature is "detected" by the Kodama observer, the energy flux should also be determined "in view of" the Kodama observer. Referring to the definition of energy-momentum tensor in standard general relativity textbooks, the 4-momentum flow measured by the Kodama observer takes the form

$$J_a = T_{ab}K^b.$$

Now the energy flux across the apparent horizon during an infinitesimal time interval $dt$ is

$$\delta Q = \int_{\mathcal{H}} J_a d\Sigma^b = \int_{\mathcal{H}} T_{ab}K^a d\Sigma^b.$$ (2.10)

Noticing that the generator (normal vector) of horizon $n^a = (\partial/\partial t)^a - Hr(\partial/\partial r)^a$, the energy flux can be calculated as

$$
\begin{aligned}
\delta Q &= AK^a T_{ab}n^b dt\Big|_{\tilde{r}=\tilde{r_A}} \\
&= A\left(K^t T_{tt}n^t + K^r T_{rr}n^r\right)\Big|_{\tilde{r}=\tilde{r_A}} \\
&= A\left(\sqrt{1-kr^2}\rho + \sqrt{1-kr^2}Hr\,p\frac{a^2}{1-kr^2}\frac{\sqrt{1-kr^2}}{a}\right)\Big|_{\tilde{r}=\tilde{r}_A} \\
&= A(\rho+p)H\tilde{r}_A.
\end{aligned}
$$ (2.11)

Finally, using the expression of $T$ and $S$ in (2.6), the Clausius relation $\delta Q = T\,dS$ implies that

$$TdS = \delta Q = A(\rho+p)H\tilde{r}_A.$$ (2.12)

Noting that

$$\dot{\tilde{r}}_A = -H\tilde{r}_A^3 \left( \dot{H} - \frac{k}{a^2} \right) \,,$$

Eq.(2.12) leads to

$$\dot{H} - \frac{k}{a^2} = -\frac{8\pi G}{n-1}(\rho + p)\,. \tag{2.13}$$

Substitute $(\rho + p)$ into (2.13) using continuity condition (2.4), and integrate, we finally get

$$H^2 + \frac{k}{a^2} = \frac{16\pi G}{n(n-1)}\rho\,,$$

which is just the Friedmann equation, the integration constant can be regarded as a cosmological constant and incorporated into the energy density $\rho$ as a special component.

One comment follows. In some articles, for example [42, 43], the energy flux is calculated directly by using the generator $n^a$ of the apparent horizon as $\delta Q = 4\pi\tilde{r}_A^2 T_{ab}n^a n^b dt$. However, this expression cannot give the correct result, so we suggest to use the expression (2.11) to calculate the energy flux $\delta Q$ in treating the apparent horizon thermodynamics in FRW model. Compared to the previous treatment, our expression not only gives the correct result, but also has significant physical meanings.

## 3. Thermodynamics of FRW universe in (IR modified) Hořava-Lifshitz gravity

In the previous section we have briefly reviewed the equivalence between Friedmann equation and the first law of thermodynamics in FRW universe. This subject has been extensively studied in various gravity theories besides Einstein theory. In Gauss-Bonnet gravity and Lovelock gravity where the entropy of black holes does not obey the area formula, the corresponding Friedmann equation can also be obtained from the apparent horizon first law of thermodynamics[5], by employing the entropy formula of static spherically symmetric black holes. However, in scalar-tensor gravity and f(R) gravity, in order to obtain the Friedmann equation, one still has to take the ansatz $T = 1/2\pi\tilde{r}_A$ and $S = A/4G$, and redefine the energy $\tilde{\rho}$ and pressure $\tilde{p}$[6]; it is also suggested to replace the original Clausius relation by a non-equilibrium one , $\delta Q = TdS + Td_iS$ in scalar-tensor theory[9] and $f(R)$ theory[11]; and it is suggested in [12] to replace the Misner-Sharp mass by a masslike function, etc. This implies that the derivations of Friedmann equation from first law of thermodynamics are subtle in various gravity theories. In this section we consider the derivation of Friedmann equation from first law of thermodynamics in the case of Hořava-Lifshitz gravity. Thermodynamics of cosmological model in Hořava-Lifshitz gravity has also been studied in [33][44]. Motivated by [6] and [33], we redefine the energy and pressure of the perfect fluid, and show that Friedmann equations can be casted into the form of the first law (2.12). Here we assume the holography ansatz $S = A/4G$, and the original Clausius relation $\delta Q = TdS$. We also study the case of IR modified Hořava-Lifshitz gravity.

The recently proposed Hořava-Lifshitz gravity may be regarded as a UV complete candidate for general relativity. The dynamic variables $N$, $N_i$, and $g_{ij}$ are given in terms

of the metric taking the ADM form[45]

$$ds^2 = -N^2 dt^2 + g_{ij} \left( dx^i + N^i dt \right) \left( dx^j + N^j dt \right) . \tag{3.1}$$

The coordinates $(t, x^i)$ scale differently with dynamical critical exponent $z = 3$,

$$\mathbf{x} \to b\mathbf{x}, \; t \to b^z t .$$

The action of Horava-Lifshitz gravity can be written as

$$\begin{aligned}
S_{HL} &= \int dt dx^i \, N \sqrt{g} \left( \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_m \right) , \\
\mathcal{L}_0 &= \frac{2}{\kappa^2} \left( K_{ij} K^{ij} - \lambda K^2 \right) + \frac{\kappa^2 \mu^2 \left( \Lambda_W R - 3\Lambda_W^2 \right)}{8(1 - 3\lambda)} , \\
\mathcal{L}_1 &= \frac{\kappa^2 \mu^2 (1 - 4\lambda)}{32(1 - 3\lambda)} R^2 \\
&\quad - \frac{\kappa^2}{2w^4} \left( C_{ij} - \frac{\mu w^2}{2} R_{ij} \right) \left( C^{ij} - \frac{\mu w^2}{2} R^{ij} \right) .
\end{aligned}$$

where the extrinsic curvature and the Cotten tensor are given by

$$\begin{aligned}
K_{ij} &= \frac{1}{2N} \left( \dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i \right) , \\
C^{ij} &= \epsilon^{ikl} \nabla_k \left( R_l^j - \frac{1}{4} R \delta_l^j \right) .
\end{aligned}$$

In the IR limit, the action should be reduced to the Einstein-Hilbert action of general relativity

$$S_{EH} = \frac{1}{16\pi G} \int d^4 x \, N \sqrt{g} \left( K_{ij} K^{ij} - K^2 + R - 2\Lambda \right) . \tag{3.2}$$

by setting $x^0 = ct$, $\lambda = 1$, the speed of light, Newton's constant, and the cosmological constant emerge as

$$c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda_W}{1 - 3\lambda}}, \; 16\pi G = \frac{\kappa^4 \mu}{8} \sqrt{\frac{\Lambda_W}{1 - 3\lambda}}, \; \Lambda = \frac{3\kappa^4 \mu^2 \Lambda_W^2}{32(1 - 3\lambda)} . \tag{3.3}$$

Taking the ansatz of cosmological solutions of the FRW metric form (2.1), for a perfect fluid matter contribution, the Friedmann equations are

$$\begin{aligned}
\frac{6}{\kappa^2} (3\lambda - 1) H^2 &= \rho - \frac{3\kappa^2 \mu^2 \Lambda_W^2}{8(3\lambda - 1)} \\
&\quad + \frac{3k\kappa^2 \mu^2 \Lambda_W}{4(3\lambda - 1)a^2} - \frac{3k^2 \kappa^2 \mu^2}{8(3\lambda - 1)a^4} , \\
\dot{\rho} + 3H(\rho + p) &= 0 .
\end{aligned} \tag{3.4}$$

The verification of temperature $T = 1/2\pi \tilde{r}_A$ via the tunneling approach in [7] and the derivation of the energy flux (2.11) through the apparent horizon are only relevant to the FRW metric ansatz, regardless of the specific gravity theories. It is reasonable to regard

them as valid in the context of Hořava-Lifshitz gravity. Furthermore, we assume that the entropy associated with the apparent horizon is also a quarter of horizon area, $S = A/4G$. Just as in [6], and motivated by [33], we define the energy and pressure of perfect fluid in the universe[33]

$$\tilde{\rho} \equiv \rho + \rho_\Lambda + \rho_k + \rho_{dr} \,, \quad \tilde{p} \equiv p + p_\Lambda + p_k + p_{dr} \,,$$

where

$$\rho_\Lambda = -p_\Lambda = -\frac{3\kappa^2\mu^2\Lambda_W^2}{8(3\lambda-1)} \,, \tag{3.5}$$

$$\rho_k = -3p_k$$
$$= \frac{3k}{4(3\lambda-1)a^2}\left(\kappa^2\mu^2\Lambda_W + \frac{8}{\kappa^2}(3\lambda-1)^2\right) \,. \tag{3.6}$$

$$\rho_{dr} = 3p_{dr} = -\frac{3\kappa^2\mu^2}{8(1-3\lambda)}\frac{k^2}{a^4} \,, \tag{3.7}$$

are the cosmological constant term, curvature term and the dark radiation term. These extra terms can be viewed as the dark components, or effective energy-momentum tensor. Identifying

$$8\pi G_{cosmo} = \frac{\kappa^2}{2(3\lambda-1)} \,, \tag{3.8}$$

and make use of the Clausius relation $\delta Q = T\, dS$, the Friedmann equation (3.4) in Hořava-Lifshitz gravity can be obtain,

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_{cosmo}}{3}\tilde{\rho} \,, \tag{3.9}$$
$$\dot{\rho}_i + 3H(\rho_i + p_i) = 0 \,.$$

Similar process can be made in the IR modified Hořava-Lifshitz gravity. The action of IR modified Hořava-Lifshitz gravity is obtained by adding a term $\mu^4 R^{(3)}$ to the original action [28],

$$S = \int dt d^3x\, N\sqrt{g}\left\{\frac{2}{\kappa^2}\left(K_{ij}K^{ij} - \lambda K^2\right) - \frac{\kappa^2}{2w^4}C_{ij}C^{ij}\right.$$
$$+\frac{\kappa^2\mu}{2w^2}\epsilon^{ijk}R_{il}\nabla_j R^l_k - \frac{\kappa^2\mu^2}{8}R_{ij}R^{ij}$$
$$\left.+\frac{\kappa^2\mu^2}{8(1-3\lambda)}\left(\frac{1-4\lambda}{4}R^2 + \Lambda_W R - 3\Lambda_W^2\right) + \mu^4 R\right\} \,. \tag{3.10}$$

By taking the FRW metric ansatz and assuming the matter contribution as perfect fluid, the Friedmann equation is[31]

$$H^2 = \frac{\kappa^2}{6(3\lambda-1)}\left(\rho - \frac{3\kappa^2\mu^2\Lambda_W^2}{8(3\lambda-1)}\right.$$
$$\left.+\frac{3k\kappa^2\mu^2\Lambda_W}{4(3\lambda-1)a^2} - \frac{6k\mu^4}{a^2} - \frac{3k^2\kappa^2\mu^2}{8(3\lambda-1)a^4}\right) \,. \tag{3.11}$$

Again, by introducing the cosmological constant term, the curvature term and the dark radiation term

$$\rho_\Lambda = -p_\Lambda = -\frac{3\kappa^2\mu^2\Lambda_W^2}{8(3\lambda-1)}\,,\tag{3.12}$$

$$\rho_k = -3p_k$$
$$= \frac{3k}{4(3\lambda-1)a^2}\left(\kappa^2\mu^2\Lambda_W - 8\mu^4(3\lambda-1) + \frac{8}{\kappa^2}(3\lambda-1)^2\right),\tag{3.13}$$

$$\rho_{dr} = 3p_{dr} = \frac{3\kappa^2\mu^2}{8(3\lambda-1)}\frac{k^2}{a^4}\tag{3.14}$$

and identifying

$$8\pi G_{cosmo} = \frac{\kappa^2}{2(3\lambda-1)}\,,\tag{3.15}$$

the first law of thermodynamics $\delta Q = T\,dS$ implies the Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_{cosmo}}{3}\tilde{\rho}\,,\tag{3.16}$$

which is just Eq.(3.11). Thus we have shown that Friedmann equations in Hořava-Lifshitz gravity and IR modified Hořava-Lifshitz gravity can both be casted into the form of first law of apparent horizon thermodynamics of FRW universe model.

## 4. Discussion and conclusion

In this article, we have verified the derivation of Friedmann equation from the Clausius relation in the FRW universe model. The Kodama vector plays an important role, both in the proof of the temperature associated with the apparent horizon via the tunneling approach, and in the calculation of the energy flux through the horizon. This indicates a physical fact that the first law of thermodynamics associated with the apparent horizon is relative to the "Kodama observer". Our derivation has significant physical meanings, and enriches the discussions of thermodynamics in the FRW universe model. In addition, we have shown that Friedmann equations in the recently proposed Hořava-Lifshitz gravity and IR modified Hořava-Lifshitz gravity can be casted into the form of first law of thermodynamics. We assume the entropy associated with the apparent horizon is a quarter of horizon area, other than the form of spherical symmetric black hole entropy in the cases of Gauss-Bonnet and Lovelock gravity. The new degrees of freedom in Hořava-Lifshitz gravity is included in the redefined energy and pressure. Our results are useful for further understanding of the holographic properties of gravity theories.

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